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Energy analysis of evaporating thin falling film instability in vertical tube

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Abstract

The Kelvin–Helmholtz instability of evaporating thin falling film flow in vertical tube is studied by method of energy analysis. Based on the rules that the interfacial capillary waves come from the balance of works done by inertial force, surface tension on phase-change interface, and also capillary force on tube wall, the stability behaviors of falling film with different Reynolds number and different perturbation wavelength are explored in detail. The analysis indicates that the main reason of film breakup by increasing tube wall heat flux is that, the stability effect of capillary adsorbability on tube wall is weakened as surface tension waving is enhanced by improving tube wall temperature. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Falling liquid film; Instability analysis; Evaporation; Flow

1. Introduction

Falling film heat transfer has long been the subject of numerous studies owing to its great importance in nuclear energy, power engineering, chemical processing, and also, in seawater desalination [1–8]. The inherent advantages of falling thin film flow are short contact time between the process fluid and heated surface, high heat flux, minimal pressure drop, minimal static head and small process fluid holdup [6]. Short contact time is a key factor endows special attraction to using falling film evaporation in MED (multi-effect distillation) seawater desalination system, which can avoid seawater scaling formed on the tube surface. High heat transfer coefficients are always desired in MED system where the temperature difference between source vapor and evaporating film is small. Absence of static head minimizes boiling point elevation of brine solution, which increases heat transfer temperature difference.

Sustaining liquid contact on the heated wall is most crucial in the application of falling liquid film. When the heated wall is no longer in intimate contact with the liquid, the safety of the evaporating system will be threatened by the rapidly rising wall temperature, as so-called the critical heat flux (CHF). This motivates the extensive concerns of instability analysis of falling film in the past decades. A number of investigators have attempted to elucidate the mechanism of the falling film flow in terms of the characteristics of the waves, the effect of surfactants and the thickness of the film based on both theoretical and experimental studies [9–14]. However, the understanding of the mechanism of falling film instability is still far from enough.

The energy analysis was introduced for film-wise condensation in inclined small/mini-diameter tube [15], and also, the correlating factors and the Kelvin–Helmlotz instability for falling condensing liquid film in vertical mini-tube with interfacial waves were reported recently by author [16]. In the present study, an analytical model employing energy consideration is formulated to explore clearly the characteristics of interfacial

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Nomenclature

A_{wave}	cross-sectional area of wave profile
c	evaporation number, $c = (k(T_w - T_S))/(\rho_l v h_{lv})$
$E(y)$	work generated or energy
F	force
h_{lv}	latent heat
k	conductivity of liquid film
q	heat flux
R	inner radius of tube or vapor constant
r	radical position, or $1/r$, the curvature
T	temperature
$\bar{u}_l$	the average velocity of liquid film
y	distance from tube wall, or $y = R - r$
z	axial position

Greek symbols

α	wave number, $\alpha = 2\pi\delta_l/\lambda_E$,
β	the angle between wavy interface and tube wall
γ	temperature coefficient of surface tension
δ_l	average thickness of liquid film
ζ	the amplitude of the waves
λ	wave length
v	viscosity
ρ	density
σ	surface tension

Subscripts

0	axial position where wave spreads to tube wall
B	Bernoulli effect
E	initial perturbation wavelength
i	phase-change interface
l	liquid
S	saturation or surface tension
v	vapor
W	tube wall

waves of evaporating falling liquid film. This analysis may be helpful for understanding the mechanism of effects on falling evaporating liquid film instability.

2. Analytical model

The physical model to be analyzed is illustrated schematically in Fig. 1. The falling liquid film is in its symmetrical laminar flow inside a vertical tube with inner radius R and with constant tube wall surface temperature T_w, to form a vertical annular flow. The tube wall temperature is higher than the saturation temperature of liquid film, T_S, so falling film can remove the heat from tube wall by free evaporation on phase-change interface. Neglecting mass loses due to evaporation, the thickness of liquid film, δ_l, can be assumed uniform within the perturbation wave length, λ_E.

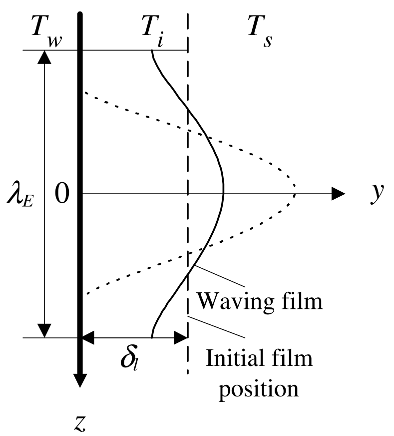

Fig. 1. Physical model.

Falling liquid film flow being instability in nature, any initial disturbance induces a waving vapor–liquid interface. Occurrence of finite amplitude capillary waves is responsible for such an appearance. The interfacial temperature will fluctuate also because of wavy interface and evaporation. This will lead to the variation of surface tension on interface, and in turn, lead to mass transfer due to thermal-capillary force. Some assumptions can be taken as: static vapor phase with constant vapor pressure, which equals to that of smooth liquid film; at an initial stage of growth, the amplitude of the waves, ζ, is equal to 0 with initial perturbation wavelength λ_E; during the growth of waves, the cross-sectional area of axially symmetric wave profile remains constant; the pressure drop from the base to the top of the liquid lump is small enough to be neglected; and the wave profile can be expressed as cosine function, i.e.,

$$y(z) = \delta_l + \zeta\cos(\alpha z/\delta_l), \quad (1)$$

where, $\alpha = 2\pi\delta_l/\lambda_E$ is the wave number.

According to the theory of Kelvin–Helmholtz instability [17], such capillary waves can be promoted in amplitude due to the suction force at the wave crest, F_B, developed as a result of the Bernoulli effect. In addition, the capillary force, F_S, caused by waving film bending in radial direction will restrain the interfacial waving. Shown as the dashed line in Fig. 1, while the waves spread to the tube wall, the enhancement of falling film instability will be restricted accordingly by capillary adhesion force, F_w, between the wall and the fluid. Thus, the interfacial waving behaviors should be established by the equilibrium of these forces. The work generated or energy involved, $E(y)$, by the capillary force, F_S, Bernoulli force, F_B, and also, the possible adhesion force on tube wall, F_w, in displacing the wave crest, can be obtained as

$$E(y) = \int_0^{y-\delta_l} [F_B(\zeta) - F_S(\zeta)]\,d\zeta, \quad \text{for } \delta_l \leqslant y < 2\delta_l \tag{2}$$

and,

$$E(y) = \int_0^{y-\delta_l} [F_B(\zeta) - F_S(\zeta)]\,d\zeta + 2\int_{\delta_l}^{y-\delta_l} F_w(\zeta)\,dz_0 \quad \text{for } y \geqslant 2\delta_l. \tag{3}$$

Among in Eq. (3), what concerned is the wave's behavior around film breakup, while the axial move of wavy interface can be neglected, so no other works by forces in axial direction, z, except for adhesion capillary force on tube wall, F_w, are included.

Taking continuous variation of the wave amplitude as disturbance of the energy system, explore the instability of evaporating falling film by energy varying caused by above-mentioned forces. As long as the total energy provided by suction force is greater than that required for working against capillary forces, $E(y)$ remains positive so that the wave crest will continue to grow up, which indicates that the system has enough energy to enhance the film instability. However, as $E(y) < 0$, the crest starts reverting and the system doesn't acquire the ability to maintain interface waving increasingly. So, $E(y) = 0$ means the minimum energy state of the system and the most possible state of the waving interface as there is no more residual energy in displacing wave crest. The parameters in Eqs. (2) and (3) can be calculated as follows:

One can obtain from Eq. (1):

$$z_0 = \frac{\lambda}{2\pi}\cos^{-1}\left(-\frac{\delta}{\zeta}\right), \tag{4}$$

where, $\lambda = \lambda_E$ and $A_{wave} = \delta_l\lambda_E$, for $\zeta \leqslant \delta_l$; as $\zeta > \delta_l$, $A_{wave} = \int_{-z_0}^{z_0} y(z)\,dz$, therefore

$$\lambda = A_{wave} \Big/ \left\{ \frac{\delta_l}{\pi}\cos^{-1}\left(-\frac{\delta_l}{\zeta}\right) + \frac{\zeta}{\pi}\sin\left[\cos^{-1}\left(-\frac{\delta_l}{\zeta}\right)\right] \right\}. \tag{5}$$

Differentiating z_0 with respect to ζ, yields

$$dz_0 = \left[\frac{1}{2\pi}\frac{d\lambda}{d\zeta}\cos\left(-\frac{\delta}{\zeta}\right) + \frac{\lambda}{2\pi}\frac{-1}{\sqrt{1-\delta^2/\zeta^2}}\left(\frac{\delta}{\zeta^2}\right) \right] d\zeta, \tag{6}$$

where, $d\lambda/d\zeta$ can be determined from differentiating Eq. (5).

Because the wavelength λ is much smaller than the tube length, the pressure drop within the respect of order of wavelength can be neglected. Also according to assumption of constant vapor pressure and initial pressure equilibrium between phases, we can induce the Bernoulli force in normal direction for one point on wavy interface from Bernoulli Equation as:

$$F_B(z,\zeta) = \frac{\rho_l}{2}(\bar{u}_l^2 - \bar{u}_l'^2) \tag{7}$$

in which, $\bar{u}_l$ is the average velocity of liquid film; and $\bar{u}_l' = \bar{u}_l \cdot [R^2 - (R-\delta_l)^2]/[R^2 - (R-y)^2]$, is the average velocity of liquid film at the radial position of $(y - \delta_l)$.

The interfacial capillary force on waving film surface can be drawn from the well-known Young–Laplace Equation as:

$$F_S(\zeta) = -\sigma_{lv}\left(\frac{1}{r_{lv,0}} + \frac{1}{R-\delta_l-\zeta}\right) \tag{8}$$

with

$$\frac{1}{r_{lv,0}} = \left.\frac{y''(z)}{[1+y'(z)^2]^{3/2}}\right|_{z=0}.$$

While the waves spread to tube wall, the adhesion force on the wall surface can be expressed as

$$F_w(\zeta) = -\sigma_{lv}\cos\beta\left(\frac{1}{r_{lw,z_0}} + \frac{1}{R}\right), \tag{9}$$

where β is the angle between wavy interface and tube wall, i.e., $tg\beta = y'|_{z=z_0}$; and

$$\frac{1}{r_{lv,z_0}} = \left.\frac{y''(z)}{[1+y'(z)^2]^{3/2}}\right|_{z=z_0}.$$

Only heat conduction in laminar liquid film is taken into consideration, the heat flux between tube wall and evaporating falling film, q, can be then obtained as

$$q = k\frac{T_w - T_i(y)}{y}, \tag{10}$$

where k is the conductivity of liquid film.

For thin falling film evaporation, the surface heat flux can be expressed as [18]:

$$q = c\left(\frac{1}{2\pi R T_S^3}\right)^{1/2}\rho_v h_{lv}^2[T_i(y) - T_S] \tag{11}$$

among which, $c = (k(T_w - T_S))/(\rho_l v h_{lv})$, is the evaporation number.

Combined with Eqs. (10) and (11), one can obtain the waving interfacial temperature as:

$$T_i(y) = \left[\frac{T_w}{y} + \frac{c\rho_v h_{lv}^2}{k} \left(\frac{1}{2\pi R T_S} \right)^{1/2} \right] \Bigg/ \left[\frac{1}{y} + \frac{c\rho_v h_{lv}^2}{k} \left(\frac{1}{2\pi R T_S^3} \right)^{1/2} \right]. \tag{12}$$

Giving a criteria temperature T_0 around the saturation temperature T_S, and also, the corresponding surface tension, σ_0, we can express the surface tension in Eqs. (8) and (9) as

$$\sigma_{lv}(y) = \sigma_0 - \gamma[T_i(y) - T_0]. \tag{13}$$

This equation embodies the influence of thermal-capillary effect on instability of evaporating falling film due to interface temperature fluctuation.

3. Numerical results with discussions

Typical numerical analysis are reported here for thin evaporating falling water film of 1 mm in initial thickness and 0.1 MPa in pressure in vertical tube of 30 mm i.d. If no special mention, the temperature drop between the vapor and tube wall surface is taken as $\Delta T = 5$ °C.

The results are illustrated in Fig. 2, in which the horizontal coordinate is the non-dimensional distance between wave crest and initial interface position, $(y - \delta_l)/\delta_l$. The two-phase flow system with smooth phase-change interface in its unsteady equilibrium. Once the interface begins to wave, the suction forces increase at first, due to the pressure drop between two phases, which promotes the waving. Subsequently, the surface tension that restrains the interfacial waves increases gradually. The combined effect of the forces tends to make the wavy interface realizing a somewhat stable, the energy involved for such process, $E(y)$ is illustrated in Fig. 2.

As shown in Fig. 2(a), at the same initial wavelength, increasing Reynolds number of film, Re, will increase film velocity. According to Eq. (7), this means more obvious effect of Bernoulli force, and in turn, results in the increases of energy involved in enhancing the phase-change interfacial instability.

The effect of initial perturbation wavelength on film instability is illustrated in Fig. 2(b). For falling film in vertical tube, there may be two converse effects of capillary force on film instability at different interface status. As $[1/r_{lv,0} + 1/(R - y|_{z=0})] < 0$, capillary force, F_S, is the factor to restrain the interfacial waves. Otherwise, F_S can be also enhance film instability. Increasing perturbation wavelength (decrease of wave number) will increase the radius of curvature at wave crest, $r_{lv,0}$, and hence, the centripetal part of capillary force becomes dominating, the instability of film increases gradually. This may indicate that those waves with longer perturbation wavelength are easier to become instability.

Fig. 2(c) shows the effect of tube wall temperature variation on film flow instability. While the interfacial waves spread to tube wall, corresponding to $(y - \delta_l)/\delta_l = 1$, the energy involved in displacing wave crest, $E(y)$, may decrease sharply because the adhesion force, $F_w(\zeta)$, restricts rising trend of the wave, which prevents the thin film from breakup. At conditions of interfacial evaporating, the waving film may also result in the fluctuant of surface tension. As shown in Eqs. (12) and (13), surface tension increases with decrease of interface temperature in wave crest and decreases with increase of interface temperature in wave trough. Improving heat transfer temperature difference, $(T_w - T_S)$, will enhance the fluctuant of interface temperature, means that the stability effect of capillary adsorbability on tube wall is weakened with the decrease of surface tension between liquid and tube wall. As shown in Fig. 2(c), higher tube wall temperature, more energy may be involved in improving interface wavy. It can be thus deduced that, increasing tube wall heat flux continuously to some extent, the system energy of promoting film waving at $(y - \delta_l)/\delta_l = 1$ may becomes positive, that is to say, liquid film breakup takes place.

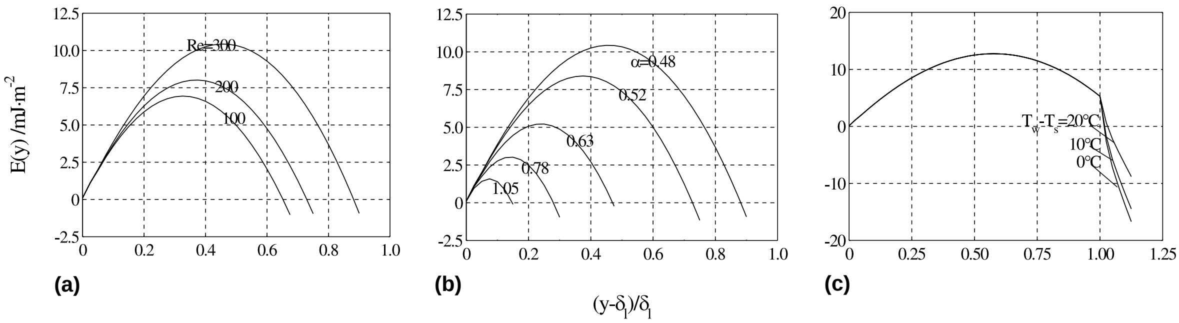

Fig. 2. Energy involved in displacing the wave crest. (a) Influence of Reynolds number, $\alpha = 0.48$. (b) Influence of wave number, $Re = 300$. (c) Influence of the capillarity on the wall surface, $Re = 200$, $\alpha = 0.40$.

4. Concluding remarks

Energy analysis has been adopted to examine the behavior of wavy interface for thin evaporating falling liquid film in a vertical tube. The effects on falling film waving of inertial force of flow, surface tension on phase-change interface and adhesion capillary force on tube wall are explored. The present study gives rational explanations to film instability enhancement due to increase of Reynolds number and perturbation wavelengths. The analysis also indicates that the main reason of film breakup by increasing tube wall heat flux is that, the stability effect of capillary adsorbability on tube wall is weakened as surface tension waving is enhanced by improving tube wall temperature.

Acknowledgements

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